Improved evaluation of the second class amplitude $\tau \to \eta\pi\nu_\tau$ in the SM

In collaboration with:

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Motivations and overview

- Second class weak decays [*Weinberg (1958)*]
  - → Violate G-parity $G = Ce^{i\pi I_y}$
  - → Standard Model: isospin beaking $m_d - m_u$, e^2
  - → Experimentally difficult to disentangle (Many β-decay of nuclei searches[*Wilkinson EPJ A7 (2000) 307*])

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- $\tau \to \eta\pi\nu_\tau$ clean second class

$$G J_\mu^{ud} G^\dagger = +J_\mu^{ud}$$
$$G|\eta\pi\rangle = -|\eta\pi\rangle$$

- Interests of $\tau \to \eta\pi\nu$ mode:

  → Suppression in SM, sensitivity to non-SM [*Bramon,Narison,Pich PL B187 (1987) 543, Nussinov, Soffer PR D78 (2008) 033006*]

  → Within SM: improved measure of $m_d - m_u$?

  → Scalar form factor $f_0^{\eta\pi}$:
  probes matrix element of scalar operator: $\langle 0|\bar{u}d|\eta\pi\rangle$
  Then: access to couplings of $a_0(980)$, $a_0(1450)$ mesons to $\bar{u}d$ ($\longrightarrow$ exotic content).

- Matrix element of vector current

$$\langle \eta\pi^+ | \bar{u}\gamma^\mu d | 0 \rangle = -\sqrt{2}\left[f_+^{\eta\pi}(s)\,(p_\eta - p_\pi)^\mu + f_-^{\eta\pi}(s)\,(p_\eta + p_\pi)^\mu \right]$$

- Scalar form factor: $f_0^{\eta\pi}(s) = f_+^{\eta\pi}(s) + \frac{s}{m_\eta^2 - m_\pi^2} f_-^{\eta\pi}(s)$

  associated with the divergence

$$\langle \eta\pi^+ | i\partial_\mu \bar{u}\gamma^\mu d | 0 \rangle = \sqrt{2}(m_\eta^2 - m_\pi^2)\, f_0^{\eta\pi}(s)$$

- Vector current not conserved! Ward identity:

$$\boxed{i\partial_\mu \bar{u}\gamma^\mu d = (m_d - m_u)\bar{u}d - eA_\mu \bar{u}\gamma^\mu d}$$

- Electro-magnetic induced amplitude
  - → Part is absorbed in $f_+^{\eta\pi}$, $f_0^{\eta\pi}$
  - → Part is not !

- Use extensively analyticity properties

1) Write dispersion relations for form factors
Example:

$$f_+^{\eta\pi}(s) = f_+^{\eta\pi}(0) + \dot{f}_+^{\eta\pi}(0)s + \frac{s^2}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\mathrm{Im}\, f_+^{\eta\pi}(s')}{(s')^2(s'-s)}ds'$$

2) Values of $f_+^{\eta\pi}(0)$, $\dot{f}_+^{\eta\pi}(0)$ from ChPT NLO [*H. Neufeld, H.Rupertsberger, ZP C71 (1996) 131*]

3) Imaginary parts from unitarity

→ $\mathrm{Im}\, f_0^{\eta\pi}$: scenario: analogy with $K\pi$ form factor

→ $\mathrm{Im}\, f_+^{\eta\pi}$: new inputs from $\eta \to 3\pi$

Results from ChPT at LO and NLO

- LO ChPT: $f_+^{\eta\pi}(s) = f_0^{\eta\pi}(s)\Big|_{LO} = \epsilon = \dfrac{\sqrt{3}(m_d - m_u)}{4\,(m_s - m_{ud})}$

$$f_+^{\eta\pi}(0)\Big|_{LO} = 0.99 \times 10^{-2} \quad \text{(PS masses +LO ChPT)}$$

$$= (1.21 \pm 0.15) \times 10^{-2} \quad \text{(lattice QCD, PDG 2012)}$$

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- NLO ChPT [*Neufeld, Rupertsberger (1995)*]. Correction=

$$\frac{2\epsilon}{3F_\pi^2(m_\eta^2 - m_\pi^2)}\Big\{64(m_K^2 - m_\pi^2)^2(3L_7 + L_8^r) - m_\eta^2(m_K^2 - m_\pi^2)L_\eta$$
$$-2m_K^2(m_K^2 - 2m_\pi^2)L_K + m_\pi^2(m_K^2 - 3m_\pi^2)L_\pi - \frac{2m_K^2(m_K^2 - m_\pi^2)}{16\pi^2}\Big\}$$
$$-\frac{2\sqrt{3}e^2 m_K^2}{27(m_\eta^2 - m_\pi^2)}\Big\{2(2S_2^r + 3S_3^r) - 9Z(L_K - \frac{1}{16\pi^2})\Big\}$$

- Remarkable relation (simple) with K_{l3}^{+}, K_{l3}^{0} decays

$$f_{+}^{\eta\pi}(0)\Big|_{LO+NLO} = \frac{1}{\sqrt{3}}\left[\frac{f_{+}^{K^{+}\pi^{0}}(0)}{f_{+}^{K^{0}\pi^{+}}(0)} - 1 - \frac{3e^{2}}{4(4\pi)^{2}}\log\frac{m_{K}^{2}}{m_{\pi}^{2}}\right]$$

- Exp. inputs from K-factories [*NA48, ISTRA+, KLOE, BNL-E865*] quite precise, yielding

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- Scalar form factor: reduced chiral corrections at $s = m_\eta^2 - m_\pi^2$ [*Dashen, Weinstein (1969)*]

$$f_0^{\eta\pi}(m_\eta^2 - m_\pi^2) = -\frac{F_\eta^{(3)}}{F_\pi} + O(m_\pi^2)$$

Scalar form factor

- Unitarity relation: $\eta\pi$ contrib. ($s < 1$ GeV^2):

$$\mathrm{Im}\, f_0^{\eta\pi}(s) = \theta(s-(m_\eta+m_\pi)^2)\frac{\sqrt{\lambda_{\eta\pi}(s)}}{32\pi s} \times f_0^{\eta\pi}(s)\int_{-1}^{1} dz\, T^*_{\eta\pi^+\to\eta\pi^+}(s,t,u) + \ldots$$

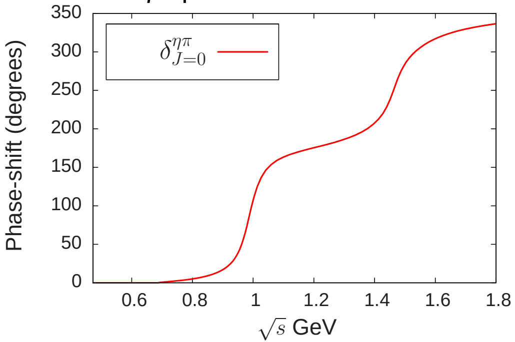
→ Phase of form factor ($\phi^{\eta\pi}$) = phase of $J=0$ scattering

→ Use phase representation (analyticity, again)

$$f_0^{\eta\pi}(s) = f_0^{\eta\pi}(0)\left(\frac{f_0^{\eta\pi}(\Delta_{\eta\pi})}{f_0^{\eta\pi}(0)}\right)^{\frac{s}{\Delta_{\eta\pi}}} \times \exp\left(\frac{s(s-\Delta_{\eta\pi})}{\pi}\int_{(m_\eta+m_\pi)^2}^{\infty} ds'\frac{\phi^{\eta\pi}(s')}{s'(s'-\Delta_{\eta\pi})(s'-s)}\right)$$

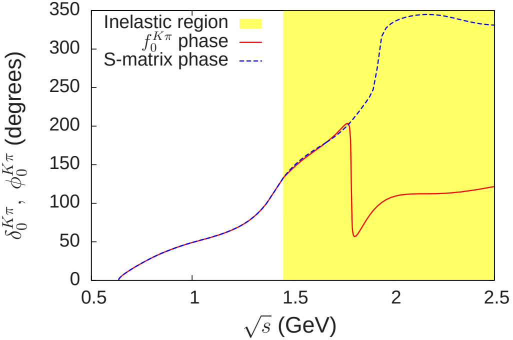
implements $s=0$, $s=m_\eta^2-m_\pi^2$ chiral constraints
[Bernard,Passemar,Oertel,Stern (2006)]

- $\eta\pi$ scattering phase: use model [*Black, Fariborz, Schechter, PR D61 (2000) 074030*]
  - → Simple: resonance exchanges + chiral constraints
  - → Similar models tested in $\pi\pi$, πK, $\eta' \to \eta\pi\pi$
  - → Result for $\eta\pi$ phase-shift:



  - → Gives us form factor phase for $s < 4m_K^2$

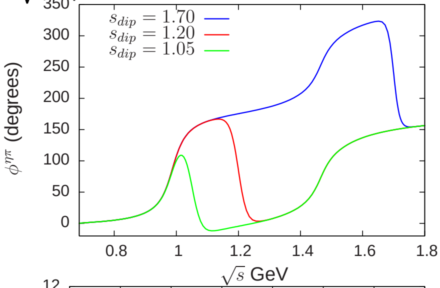
- Form factor phase $s > 1$ GeV: use analogy with $K\pi$
  - → Experimental info available on elastic and inelastic scattering (dominated by $K\eta'$)
  - → Phase from solving coupled Muskhelishvili-Omnès eqs. [*Jamin, Oller, Pich (2001)*]



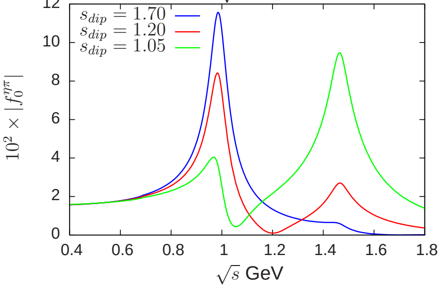
  - → phase shows sharp dip
  - → Note: $\lim_{s\to\infty} \phi^{K\pi} = \pi$ gives $f_0^{K\pi}(s) \sim 1/s$

- Assume similar dip for $\eta\pi$ form factor phase
  - → Vary dip position: $\sqrt{s_{dip}} = 1.05,\ 1.20,,\ 1.70$ GeV:

Phase:



Modulus:



Vector form factor

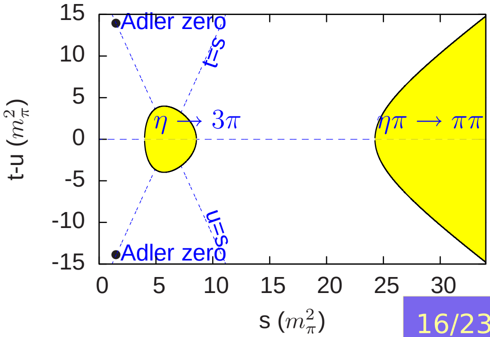
- Unitarity relation: $\pi\pi$ contribution dominates:

$$\text{disc}[f_+^{\eta\pi}(s)]\Big|_{\pi\pi} = -\theta(s-4m_\pi^2)\times \frac{s-4m_\pi^2}{32\pi\sqrt{\lambda_{\eta\pi}(s)}} F_V^\pi(s) \times \int_{-1}^{1} dz z T^*_{\pi^0\pi^+\to\eta\pi^+}(s,t)$$

other contributions: $\pi\eta$, 4π, $K\bar{K}$ either vanishing or suppressed below 1 GeV

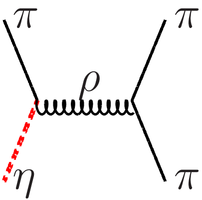
→ F_V^π is pion form factor

→ $\pi^0\pi^+ \to \eta\pi^+$ same function as $\eta \to \pi^0\pi^+\pi^-$, different region of s, t, u variables



- Approximations to $(\eta\pi \to \pi\pi)_{J=1}$

  1) ρ resonance in one channel:



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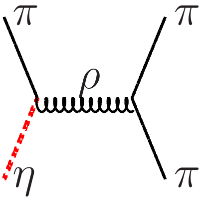
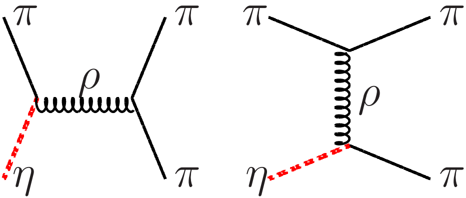


2) ρ in all channels:

[*S. Nussinov,A. Soffer, (2008)*]



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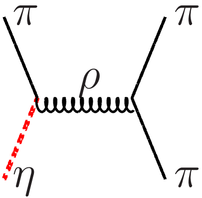

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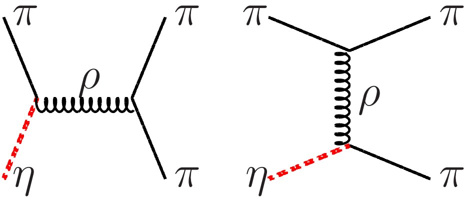
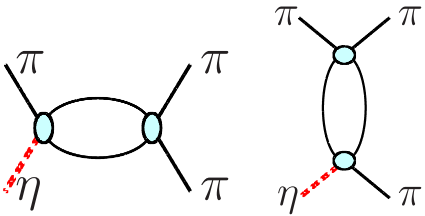


  3) ρ + scalar (σ) resonances ?

- Use Khuri-Treiman formalism (recently: [*J. Kambor et al. (1995), A. Anisovich, H. Leutwyler (1996)*])



- Family of solutions (AL treatment) with 4 parameters
  - → AL proposal: parameters/ϵ from matching ChPT NLO (near Adler zero). Then, determine ϵ from $\eta \to 3\pi$ rate
  - → But: Dalitz parameters not well reproduced
  - → Our approach: use only Adler zero from ChPT, use $\epsilon = (1.21 \pm 0.15)\,10^{-2}$ from PDG, fit parameters to exp.

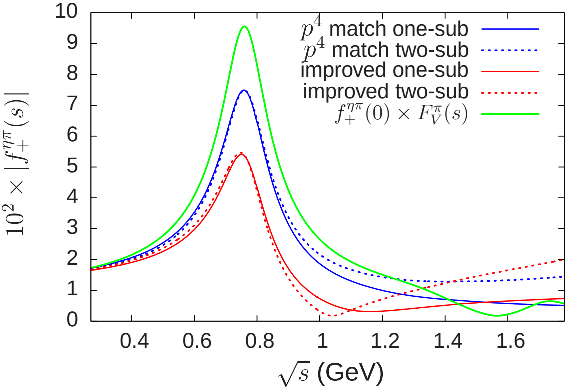
- Dalitz plot parameters of charged mode $\eta \to \pi^+\pi^-\pi^0$ [*KLOE, JHEP* ***0805*** *(2008) 006*]

param.	experimental	NLO Match.	Fit
a	$-1.090 \pm 0.005^{+0.008}_{-0.019}$	-1.300	-1.065
b	$0.124 \pm 0.006 \pm 0.010$	0.463	0.159
d	$0.057 \pm 0.006^{+0.007}_{-0.016}$	0.069	0.066
f	$0.14 \pm 0.01 \pm 0.02$	0.001	0.107

- Dalitz plot parameters of neutral mode $\eta \to \pi^0\pi^0\pi^0$ [*CRYSTAL BALL@BNL (2001), MAMI-B (2009),MAMI-C (2009), WASA@COSY (2009), KLOE(2010)*]

α	-0.0315 ± 0.0015	0.015	-0.0355

Result for $f_+^{\eta\pi}$

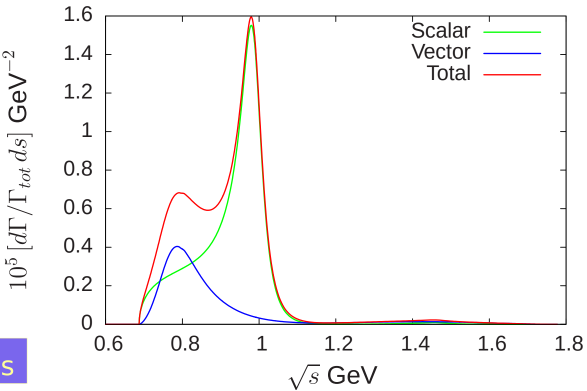


- → All curves with same $f_+^{\eta\pi}(0)$
- → Naive Breit-Wigner larger than disp. calculations
- → Once and twice subtracted disp. close $s < 1$ GeV^2

- $\tau \to \eta\pi\nu_\tau$ decay
  - → Spectral function:

$$\frac{d\Gamma}{ds} = \frac{G_F^2 V_{ud}^2 S_{EW}\, m_\tau^3}{384\,\pi^3} \frac{\sqrt{\lambda_{\eta\pi}(s)}}{s^3} \left(1 - \frac{s}{m_\tau^2}\right)^2 \times \left\{ |f_+^{\eta\pi}(s)|^2 \lambda_{\eta\pi}(s) \left(1 + \frac{2s}{m_\tau^2}\right) + 3|f_0^{\eta\pi}(s)|^2 \Delta_{\eta\pi}^2 \right\}$$

  - → Our results



- Branching fraction of $\tau \to \eta\pi\nu$

Our estimate:

$$BF_{vect} \simeq 0.11 \times 10^{-5}$$

$$BF_{scal} \simeq 0.37^{+0.30}_{-0.20} \times 10^{-5}$$ (variation of s_{dip})

is on the low side of previous ones : ($\times 10^5$)

V	S	total	ref.
0.25	1.60	1.85	Tisserant, Truong (1982)
0.12	1.38	1.50	Pich (1987)
0.15	1.06	1.21	Neufeld, Rupertsberger(1995)
0.36	1.00	1.36	Nussinov, Soffer (2008)
[0.2-0.6]	[0.2-2.3]	[0.4-2.9]	Paver, Riazuddin (2010)

Exp.: $BF \leq 9.9 \times 10^{-5}$ [*Babar (2011)*]

Conclusions

- Beyond naive Breit-Wigner using analyticity
- Scalar form factor: analogy with $K\pi$ suggests dip
- Remark: $K\pi$ scalar form factor can be probed via angular distribution in $\tau \to K\pi\nu_\tau$ decays
- Vector form factor: new experimental constraints from $\eta \to \pi^+\pi^-\pi^0$ used
- Rate suggested to be in the lower range of previous estimates
- Part of photon-loop induced amplitude not yet included