LATTICE DATA AND THE τ V_{us} PUZZLE

KM, P. Boyle, L. Del Debbio, N. Garron, R. Hudspith, E. Kerrane, J. Zanotti

Tau'12, Nagoya, September 17, 2012

OUTLINE

- *Background:* V_{us} *from FB* τ, τ*-EM sum rules*
- *OPE c.f. lattice results for the relevant correlators*
- *Lattice-lesson-motivated preliminary* V_{us} *updates*

THE V,A CORRELATORS

- Objects of interest: $ij = ud, us$, $J = 0, 1$ $\Pi^{(J)}_{ij;V/A}(Q^2)$

- Minkowski space:

$$\Pi^{\mu\nu}_{V/A}(q^2) \equiv i \int d^4x \, e^{iq\cdot x} \langle 0| T \left(J^{\mu}_{V/A}(x) J^{\dagger\,\nu}_{V/A}(0) \right) |0\rangle$$

$$= \left(q^\mu q^\nu - q^2 g^{\mu\nu} \right) \Pi^{(1)}_{V/A}(q^2) + q^\mu q^\nu \, \Pi^{(0)}_{V/A}(q^2)$$

- Euclidean space:

$$\Pi^{\mu\nu}_{V/A}(Q^2) = \left(Q^2 \delta^{\mu\nu} - Q^\mu Q^\nu \right) \Pi^{(1)}_{V/A}(Q^2) - Q^\mu Q^\nu \, \Pi^{(0)}_{V/A}(Q^2)$$

- EM V, $ij = ud, us$ V,A spectral functions, ρ_{EM}, $\rho^{(J)}_{ij;V/A}$, accessible from $\sigma[e^+e^- \rightarrow hadrons]$, hadronic τ decay data ratio $R_{ij;V/A} \equiv \frac{\Gamma[\tau \rightarrow \nu_\tau \, \text{hadrons}_{ij;V/A}\,(\gamma)]}{\Gamma[\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e(\gamma)]}$

$$\sigma_{bare}(s) = \frac{16\pi^3 \alpha_{EM}(0)^2}{s} \rho_{EM}(s)$$

$$\frac{dR_{V/A;ij}}{ds} = \frac{12\pi^2 |V_{ij}|^2 S_{EW}}{m_\tau^2} \left[w_\tau \left(\frac{s}{m_\tau^2} \right) \rho^{(0+1)}_{ij;V/A}(s) + w_L \left(\frac{s}{m_\tau^2} \right) \rho^{(0)}_{ij;V/A}(s) \right]$$

- The τ FB V_{us} sum rule:

  * $\delta R_\tau \equiv \frac{R_{ud;V+A}}{|V_{ud}|^2} - \frac{R_{us;V+A}}{|V_{us}|^2}$

  * Basic FESR relation (Cauchy's Theorem), valid for any $s_0 > 0$, analytic $w(s)$, $\Pi = \Pi^{(0+1)}_{ij;V/A}$ or $s\,\Pi^{(0)}$:

$$\int_0^{s_0} ds\, w(s)\, \rho(s) = \frac{-1}{2\pi i} \oint_{|s|=s_0} ds\, w(s)\, \Pi(s)$$

  * OPE representation $[\delta R_\tau]^{OPE}$ (begins at $D=2$, few % of separate ud, us terms) $\Rightarrow$

$$|V_{us}| = \sqrt{\frac{R_{us;V+A}}{\frac{R_{ud;V+A}}{|V_{ud}|^2} - [\delta R_\tau]^{OPE}}}\ .$$

* Details/complications:

  - $D=2$, $J=0$ $[\delta R_\tau]^{OPE}$ series out of control, badly violates spectral positivity constraints $\Rightarrow$ must subtract $J=0$ experimental dR/ds contributions

  - χ'lly unsuppressed π, K terms dominate subtraction; small χ'lly suppressed continuum us scalar, PS contributions via "highly constrained", mildly model-dependent phenomenological approaches

  - Advantage: subtraction yields $\rho_{ij;V/A}(s)^{(0+1)}$, generalized $J=0+1$ V_{us} FESRs with arbitrary $w(s)$, s_0, based on the correlator difference

$$\Delta\Pi_\tau \equiv \Pi^{(0+1)}_{ud;V+A} - \Pi^{(0+1)}_{us;V+A}$$

(consistency checks via $w(s)$-, s_0-independence)

- Conventional $\Delta\Pi_\tau$-based determination with $s_0 = m_\tau^2$, recent us BR updates [A. Adametz July 2011 BaBar thesis], 4-loop, Adler function, CIPT $D = 2$ evaluation, yields

$$|V_{us}| = 0.2176(25)_{exp}(10??)_{th}$$

$\sim 3\sigma$ below 3-family unitarity expectations, K physics results

- Residual theory issue: slow convergence of the $J = 0 + 1$ $D = 2$ series (see below)

- Related residual theory issue: FOPT vs CIPT (fixed scale vs "local scale") for $D = 2$ OPE integrals: significant difference in associated V_{us}

- The mixed τ-EM FB V_{us} sum rule:

  * FESRs involving $\rho^{(0+1)}_{ud,us;V/A}$, ρ_{EM} and the FB correlator combination

$$\Delta\Pi_{\tau-EM} \equiv 9\Pi_{EM} - 5\Pi^{(0+1)}_{ud;V} + \Pi^{(0+1)}_{ud;A} - \Pi^{(0+1)}_{us;V+A}$$

  constructed to kill LO coefficient in leading $D = 2$ OPE series

  * Strong $D = 2$ OPE suppression (by construction); $D = 4$ suppression for free

  * Suppression not due to hidden symmetry as VSA version of $D = 6$ not suppressed

* $D = 2, 4$ contributions, $\bar{a} = \frac{\alpha_s(Q^2)}{\pi}$, $\bar{m}_s = m_s(Q^2)$

$$\left[\Delta\Pi_\tau(Q^2)\right]^{OPE}_{D=2} = \frac{3}{2\pi^2}\frac{\bar{m}_s^2}{Q^2}\left[1 + 2.333\bar{a} + 19.933\bar{a}^2 + 208.746\bar{a}^3 + \cdots\right]$$

$$\left[\Delta\Pi_{\tau-EM}(Q^2)\right]^{OPE}_{D=2} = \frac{-3}{2\pi^2}\frac{\bar{m}_s^2}{Q^2}\left[0 + \frac{1}{3}\bar{a} + 4.3839\bar{a}^2 + 44.943\bar{a}^3 + \cdots\right]$$

$$\left[\Delta\Pi_\tau(Q^2)\right]^{OPE}_{D=4} = \frac{\left[\langle m_\ell\bar{\ell}\ell\rangle - \langle m_s\bar{s}s\rangle\right]}{Q^4}\left(2 - 2\,\bar{a} - \frac{26}{3}\,\bar{a}^2\right)$$

$$\left[\Delta\Pi_{\tau-EM}(Q^2)\right]^{OPE}_{D=4} = \frac{\left[\langle m_\ell\bar{\ell}\ell\rangle - \langle m_s\bar{s}s\rangle\right]}{Q^4}\left(0 - \frac{8}{3}\bar{a} - \frac{59}{3}\bar{a}^2\right)$$

* The $\Delta\Pi_\tau$ Determination: $a(m_\tau^2) \sim 0.1$ hence $D = 2$

$\Delta\Pi_\tau$ convergence issue (series looking asymptotic at spacelike point for *all* kinematically accessible s_0)

- For the CIPT prescription, $D = 2$ $\Delta\Pi_\tau$ convergence improves on moving away from the spacelike point on the contour (via the running of $\alpha_s(Q^2)$) but the effect is, in fact, minor at the correlator level, and for the weight w_τ

- Cancellations from integration on the contour at 4- and 5-loop order create misleading impression

- Slow convergence reflected in $\sim$ 0.0020 difference in $|V_{us}|$ from FOPT or CIPT for $D = 2$ at 4-loop and (estimated) 5-loop truncation orders

* For the $\Delta\Pi_{\tau-EM}$ $|V_{us}|$ determination:

  - Vanishing of LO coefficient in the $D = 2, 4$ $\Delta\Pi_{\tau-EM}$ series, suppression of remaining $D = 2$ ones c.f. $\Delta\Pi_\tau$ $\Rightarrow$ role for τ-EM analogue of $[\delta R_\tau]^{OPE}$ in $\Delta\Pi_{\tau-EM}$-based $|V_{us}|$ determination *expected* to be much reduced, suppress theoretical uncertainty

  - Price to pay is enhancement of experimental errors (no cancellation of τ, EM normalization uncertainties; impact of ud V, EM cancellation)

  - Issue for the τ-EM FESRs: Strong suppression of $\Delta\Pi_{\tau-EM}$ c.f. $\Delta\Pi_\tau$ real or just an artifact of remaining few low-order expansion terms after deliberate suppression of LO coefficient?

LATTICE DATA c.f. $[\Delta\Pi_\tau]^{OPE}$, $[\Delta\Pi_{\tau-EM}]^{OPE}$

- RBC/UKQCD data for $\Delta\Pi_\tau(Q^2)$, $\Delta\Pi_{\tau-EM}(Q^2)$

- Generated using new $32^3\times 64^\times 32_5$ Iwasaki+DSDR DWF configurations with near-physical m_π [248 and 171 MeV]

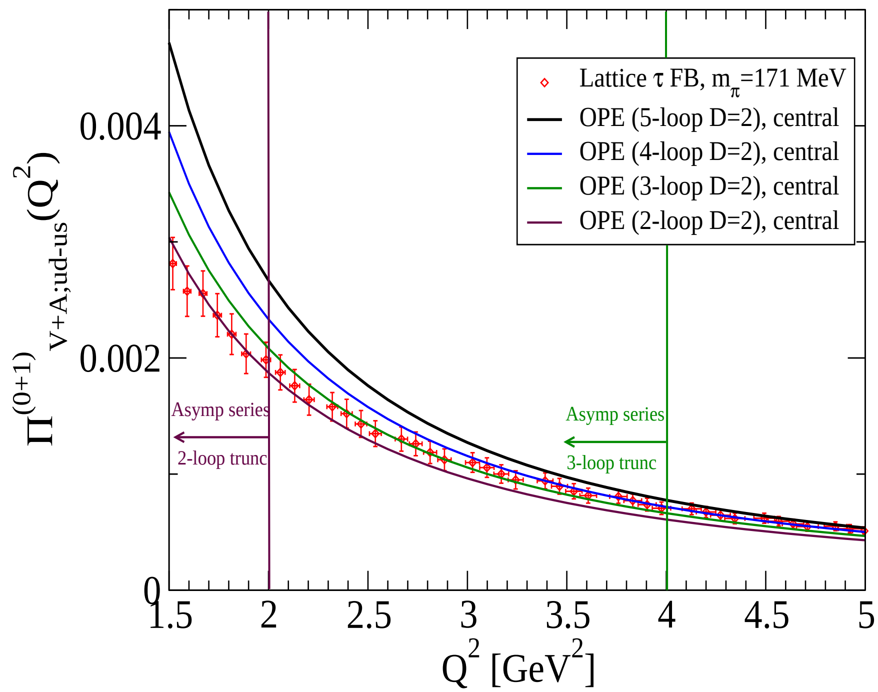
- $1/a = 1.37$ GeV, $m_\pi L \sim 5.8$ ($m_\pi = 248$ MeV), ~ 4.0 ($m_\pi = 171$ MeV)

- *For simulation details see arXiv:1208.4412, hep-lat*

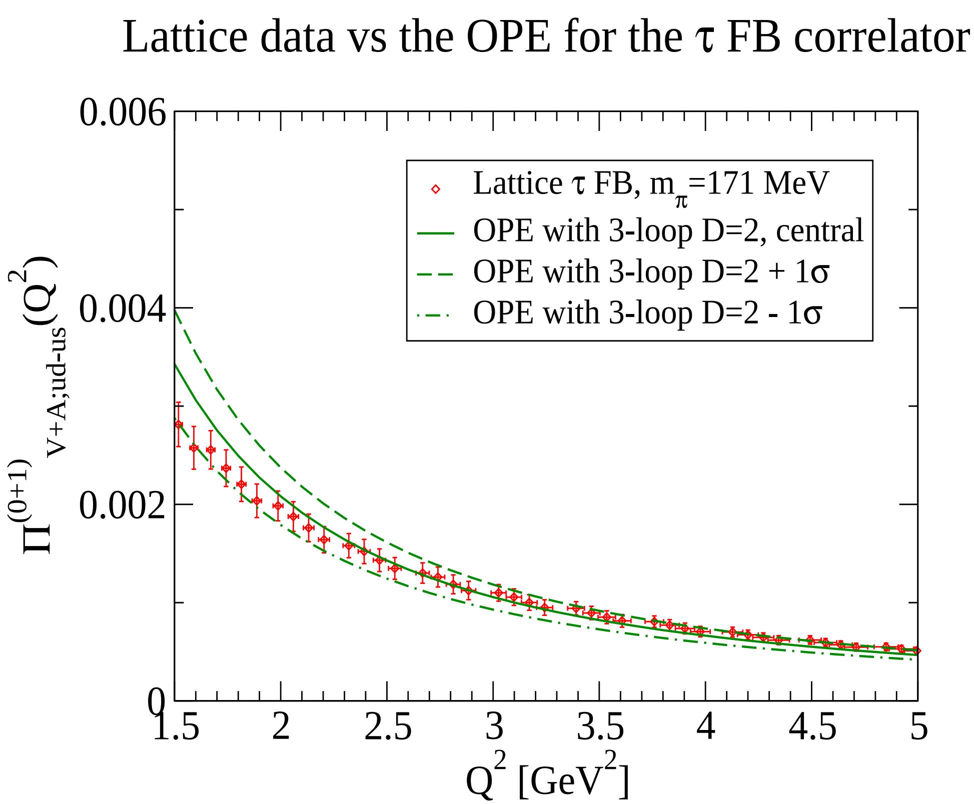
- Allows exploration of issues/questions above for spacelike-Q^2 (lattice c.f. OPE with lattice m_q, f_π, m_π)

Lattice data vs $\left[\Delta\Pi_\tau(Q^2)\right]^{OPE}$, various $D=2$ truncations

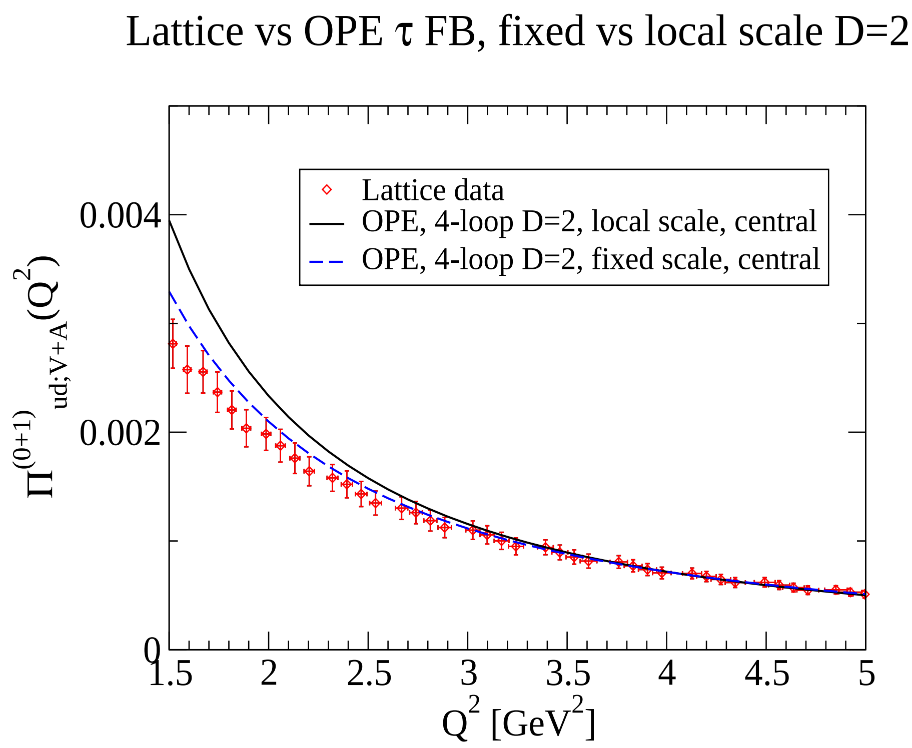
Lattice data vs the OPE for the τ FB correlator



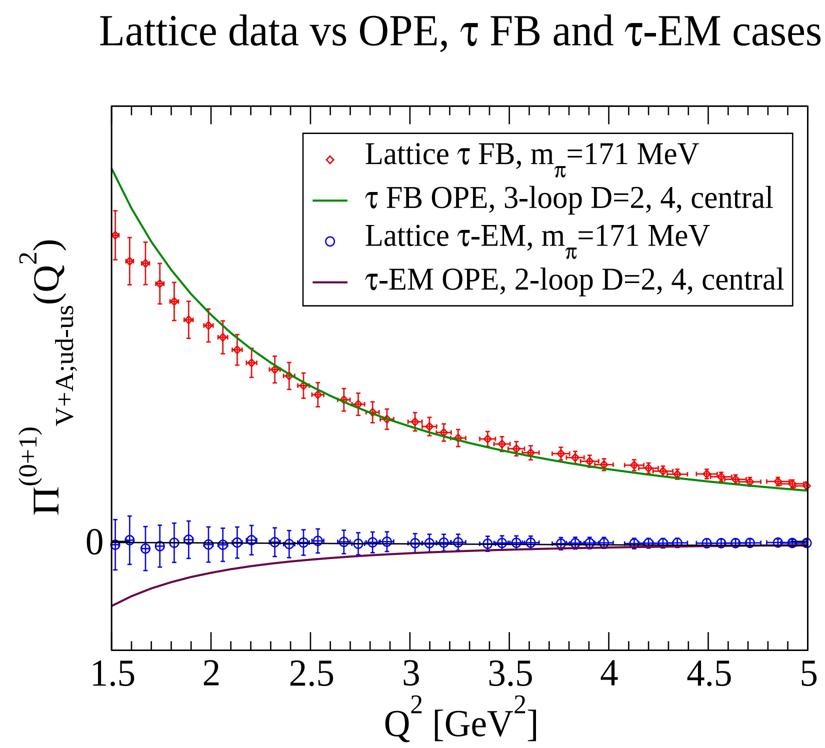
OPE error: lattice data vs $\left[\Delta\Pi_\tau(Q^2)\right]^{OPE}$, 3-loop $D=2$



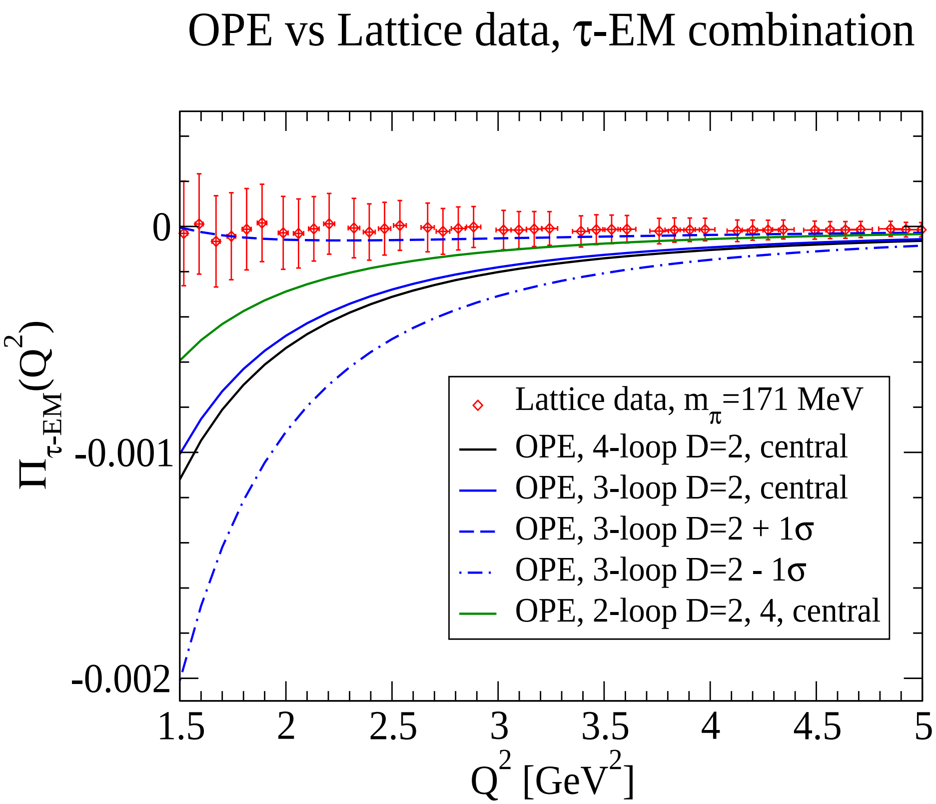
CIPT or FOPT? Local- vs fixed-scale $D = 2$ for $\left[\Delta\Pi_\tau(Q^2)\right]^{OPE}$



Actual vs nominal-OPE suppression of $\Delta\Pi_{\tau-EM}(Q^2)$ c.f. $\Delta\Pi_{\tau}(Q^2)$



Lattice data vs $\left[\Delta\Pi_{\tau-EM}(Q^2)\right]^{OPE}$



TENTATIVE LESSONS FROM THE LATTICE DATA

- OPE correlator series behaving "asymptotically", despite slow convergence to minimum term (4-loop, estimated 5-loop $D = 2$ terms *worsen* agreement with lattice $\Delta\Pi_\tau(Q^2)$)

- 3-loop $D = 2$ provides good $\Delta\Pi_\tau(Q^2)$ OPE representation for Q^2 between $\sim 2\ GeV^2$ and m_τ^2

- $D = 2$ truncation: fixed-scale favored over local-scale (suggests FOPT over CIPT for FESR integrals)

- Strong suppression of $\Delta\Pi_{\tau-EM}$ c.f. $\Delta\Pi_\tau$ confirmed (even stronger than OPE version)

PRELIMINARY $|V_{us}|$ FOR PRESCRIPTIONS FAVORED BY LATTICE DATA

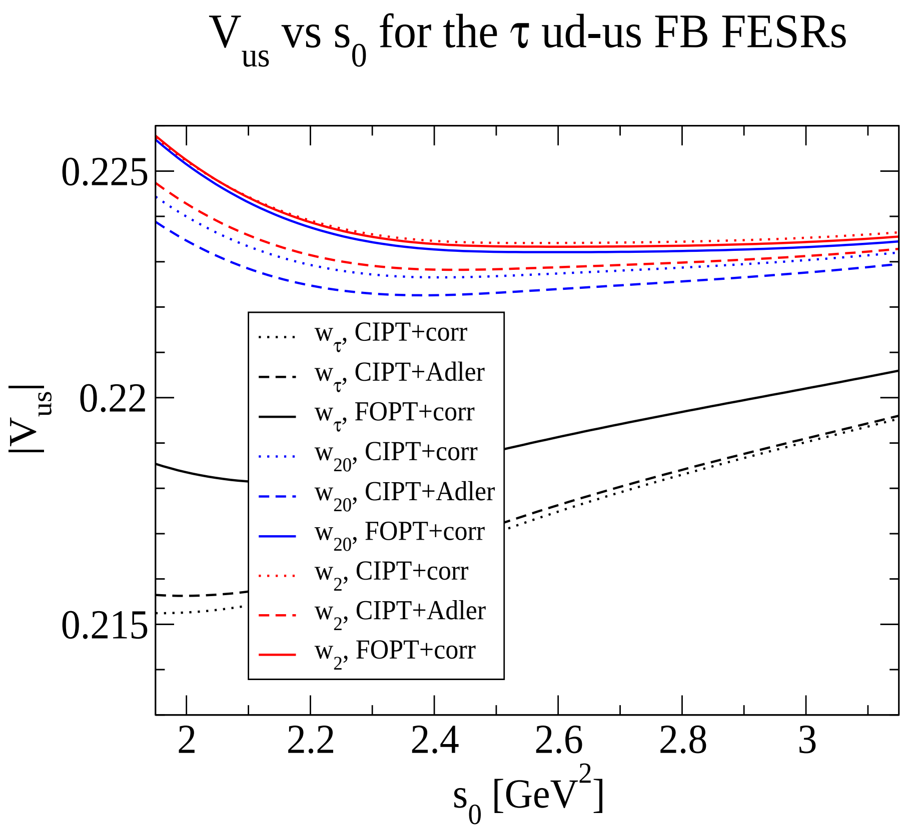
- Results for $\Delta\Pi_\tau$-based FESRs using FOPT or CIPT for $D = 2$, CIPT in either correlator or Adler function form, same order truncation

- Results for $\Delta\Pi_{\tau-EM}$-based FESRs using either 2-loop-truncated $D = 2, 4$ (best match to lattice data, though still too large) or ignoring OPE contributions entirely

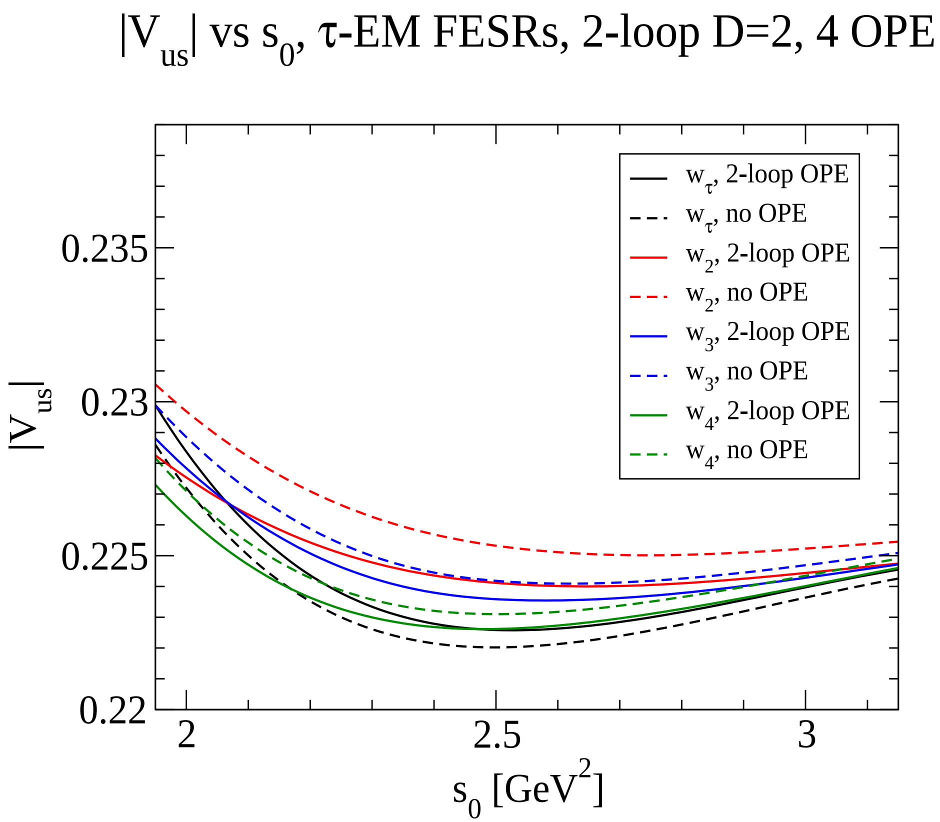
- In both cases, show results for a range of $w(s)$ and over the range 2 $GeV^2 < s_0 < m_\tau^2$

- Details on updating of τ ud V, A data, treatment of us V+A data, EM cross-sections elsewhere

$|V_{us}|$ vs s_0 from the $\Delta\Pi_\tau(Q^2)$ FESRs



$|V_{us}|$ vs s_0 from the $\Delta\Pi_{\tau-EM}(Q^2)$ FESRs



CONSTRAINTS ON THE π', π'' DECAY CONSTANTS

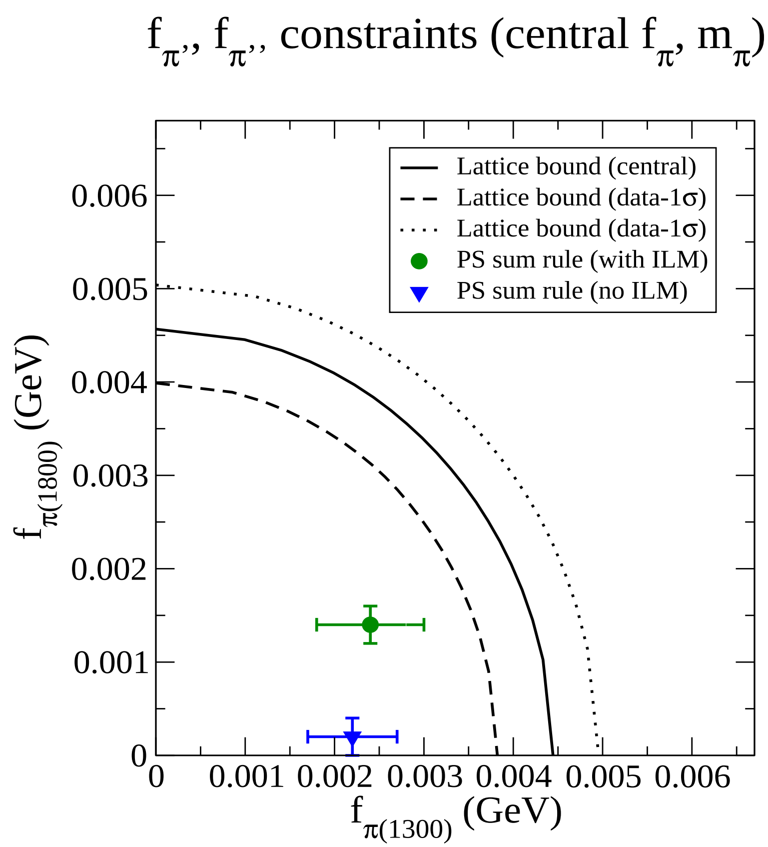
- Basic idea: Rearranged, once-subtracted dispersion relation for $P(Q^2) \equiv Q^2\,\Pi^{(0)}_{V-A}(Q^2) = -Q^2\,\Pi^{(0)}_A(Q^2)$ (constraints on χ'lly suppressed excited PS state decay constants from quantities measurable on the lattice):

$$P(Q^2) - P(Q_0^2) + \frac{(Q^2 - Q_0^2)\,2f_\pi^2 m_\pi^2}{(s+Q^2)(s+Q_0^2)} =$$
$$-\,(Q^2 - Q_0^2)\int_{9m_\pi^2}^{\infty} ds\,\frac{s\,\rho_A^{(0)}(s)}{(s+Q^2)(s+Q_0^2)}$$

- $\rho_A^{(0)} > 0 \Rightarrow$ individual resonance constraints

- Scaling to physical m_q: linearity of $f_{\pi'}, f_{\pi''}$ with $m_u + m_d$

$m_\pi = 289$ MeV, $1/a = 2.28$ GeV lattice constraints scaled down to physical m_q



SUMMARY

- For the $\Delta\Pi_\tau$ V_{us} Determination:

  * Convergence problem confirmed for $\Delta\Pi_\tau$ $D = 2$ series; 3-loop truncation favored for $Q^2 \sim 2\ GeV^2 \to m_\tau^2$

  * Conventional $\Delta\Pi_\tau$ determination does not yield s_0-independent $|V_{us}|$ below $s_0 = m_\tau^2$

  * Reduced (~ 0.0010) FOPT-CIPT difference for $|V_{us}|$ with 3-loop $D = 2$ truncation

  * Significant $w(s)$-dependence remains in V_{us} results

  * Conclusion: previous theory errors significantly underestimated

- For the $\Delta\Pi_{\tau-EM}$ V_{us} Determination:

  * Strong suppression of $\Delta\Pi_{\tau-EM}$ c.f. $\Delta\Pi_\tau$ suggested by OPE confirmed (in fact, even stronger than implied by the OPE with current central NP input)

  * Improved s_0- and $w(s)$-choice-stability for V_{us}

  * Central $s_0 \sim m_\tau^2$ V_{us} results in good agreement with expectations from 3-family unitarity, K physics

  * Residual s_0-instability still to be understood (4π experimental sector a possible candidate)

- us PS $J = 0$ subtraction results fully compatible with lattice constraints

- Future Prospects:

  - * Improvements to electroproduction cross-sections in progress, VEPP-2000 especially useful for $E \simeq 1.4 \rightarrow 2.0$ GeV

  - * us spectral integrals still based on rescalings of (now-ancient) ALEPH distribution; some exclusive mode us distributions already available, others in progress: More exclusive, and, even better, full inclusive us distribution from Belle, BaBar highly desirable!

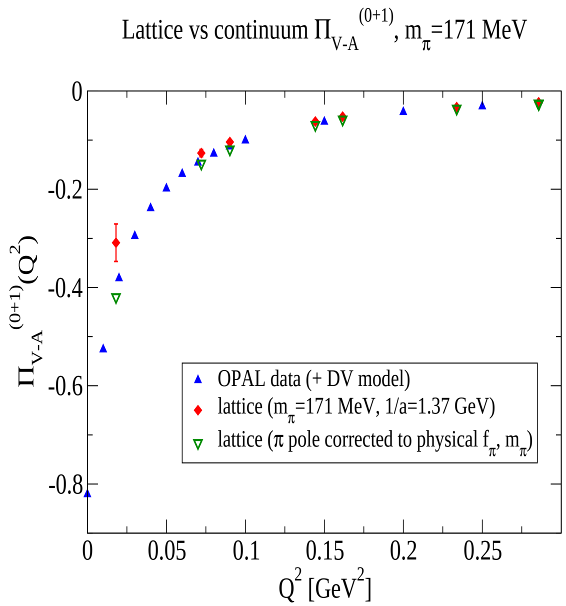
  - * Updated ud V and A distributions from Belle and BaBar also most welcome, including 4π contributions (where unexpectedly large CVC violations still not resolved)

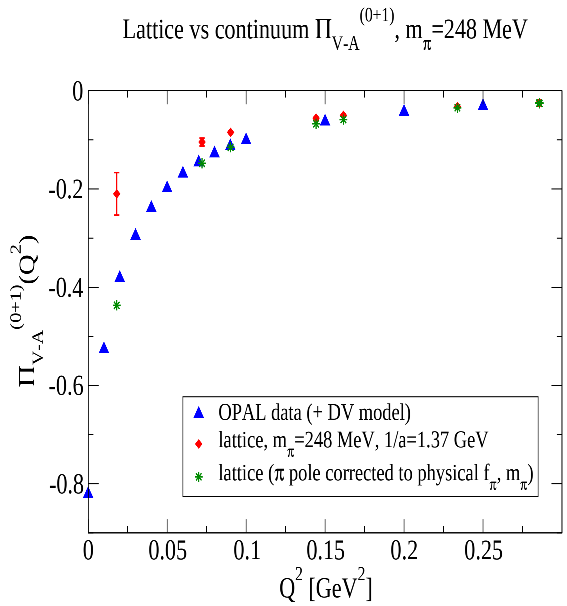
BACKUP SLIDES

- Reliability of the lattice data: comparison to continuum "data" for the $ij = ud$, V-A case

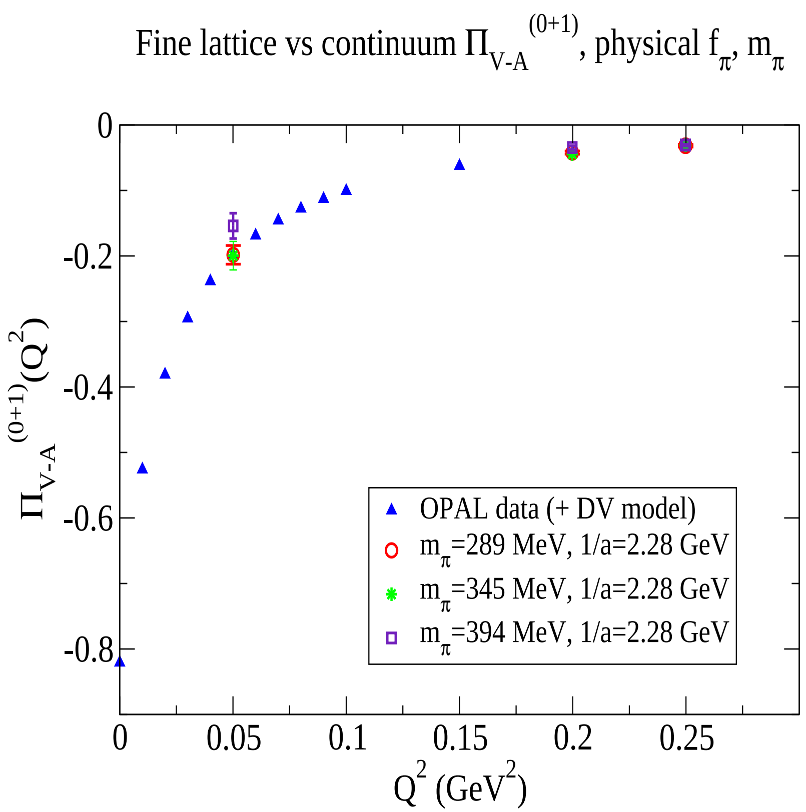
- Details of CIPT improvement on the contour for $w_\tau(y)$

Lattice data c.f. continuum $\Pi_{V-A}^{(0+1)}(Q^2)$





"Continuum" vs π-pole-corrected, $m_\pi = 289, 345, 394$ MeV



Re CIPT improvement for the $w_\tau(y)$ FESR